

# 1.2

## Divisibility by 3 and 9

### GOAL

Apply divisibility rules to determine if 3 or 9 is a factor of a whole number.

### LEARN ABOUT the Math

In the game Divide and Conquer, players use a spinner to create a four-digit number. They have 1 min to identify as many factors as possible.



How can you show that the divisibility rules for 3 and 9 make sense?

I found a shortcut online: if the sum of the digits in a number is divisible by 3 or 9, then so is the number.

Why does that work?

Sarah: 2523

Denis: 7352

## Example 1 | Investigating divisibility rules for 3 and 9

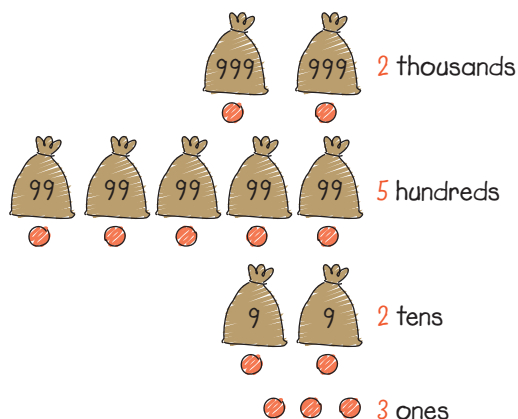


Without dividing the entire number, decide if 3 or 9 is a factor of the number.

### Sarah's Solution



I thought of my number as 2523 counters in a case.



I looked at the place values, and then put as many counters as I could into bags that hold 999, 99, or 9.

The number of counters in each bag is divisible by 3 and by 9. If the counters left over can be put in groups of 3 or 9, then the number of counters left over is also divisible by 3 or 9. This means that the total number of counters is divisible by 3 or 9 too.

I have  $2 + 5 + 2 + 3 = 12$  ones left over.

12 is also the sum of the digits of 2523!

12 is divisible by 3, but not by 9, so 2523 is divisible by 3, but not by 9.



### Denis's Solution

I renamed my number, 7352, as 7 thousands + 3 hundreds + 5 tens + 2 ones.

If I divide by 3 or 9:

7 thousands will have 7 left over.

3 hundreds will have 3 left over.

5 tens will have 5 left over.

2 ones will have 2 left over.

Each thousand will have 1 left over when I divide it by 3 or 9.

$$\begin{array}{r} 333 \text{ R } 1 \\ 3 \overline{)1000} \end{array}$$

$$\begin{array}{r} 111 \text{ R } 1 \\ 9 \overline{)1000} \end{array}$$

So will hundreds and tens.



Total left over:  $7 + 3 + 5 + 2 = 17$ .  
 $17 \div 3 = 5 \text{ R}2$

17 and 7352 have a remainder of 2 when divided by 3, so 7352 is not divisible by 3. This means that it can't be divisible by 9 either.

### Reflecting

- A. How did Denis know that 7352 is not divisible by 9?
- B. If a number is divisible by 9, is it always divisible by 3? Explain.
- C. Do the divisibility rules for 3 and 9 work for numbers with more than four digits? Explain.

## WORK WITH the Math



### Example 2

### Applying divisibility rules for 3 and 9

Use divisibility rules to determine the remainder when you divide 4434 by 3 or 9.

### Megan's Solution

$$4 + 4 + 3 + 4 = 15$$

$$15 \div 3 = 5 \text{ R}0$$

$$15 \div 9 = 1 \text{ R}6$$

$4434 \div 3$  has remainder 0.

$4434 \div 9$  has remainder 6.

I calculated the sum of the digits.

The number and the sum of the digits have the same remainder when you divide by 3 or 9.

### A Checking

1. Use divisibility rules to determine if 3 or 9 is a factor of each number. If 3 or 9 is not a factor, explain how to use the rules to determine the remainder.
  - a) 657
  - b) 3278
  - c) 4002
  - d) 37 782

2. Each number below is divisible by 9. Identify all the possibilities for the missing digit.
- a) 533■      b) ■678      c) 30■6      d) 3■27



## **B** Practising

3. Use a divisibility rule to solve each problem.
- a) 1117 trees are planted in 9 rows. Is it possible for each row to have an equal number of trees?
- b) A dairy has 5529 kg of cheese to be made into 3 kg packages. Will any cheese be left over?
4. Use divisibility rules to determine if 3 or 9 is a factor of each number. If 3 or 9 is not a factor, explain how to use the rules to determine the remainder.
- a) 6957      b) 5273      c) 44 442      d) 68 073
5. What are the greatest and least four-digit numbers that are divisible by 9?
6. A three-digit number is divisible by 10, and its middle digit is 5. Can you determine what the number is if you know that it is also divisible by 3? What if it is divisible by 9? Show your work.
7. Choose any four-digit number. Rearrange its digits to create another four-digit number. Explain why both numbers have the same remainder when you divide by 9.
8. Create a four-digit number for each description. Explain how you created one of the numbers.
- a) a number not divisible by 3 and not divisible by 2
- b) an even number divisible by 9
- c) a number divisible by 3 and 10
- d) a number divisible by 9 and 5
9. What is the least number that is greater than 3876 and divisible by 3? Explain your strategy.
10. Would you rather divide 18 927 and 17 658 by 9, or use a divisibility rule to see if 9 is a factor? Justify your choices.
11. How are the divisibility rules for 3 and 9 the same as the divisibility rules for 2, 5, and 10? How are they different?

### **Reading Strategy**

Read the problem in question 6. What inferences can you make? How can they help you solve the problem?